MATH 320 Unit 1 Exam Name:

Instructions:

Please write your answers on separate paper. Please write clearly and legibly, using a large font and plenty of white space (I need room to put my comments). Staple all your pages together, with your problems in order, when you turn in your exam. Make clear what work goes with which problem. Put your name or initials on every page. To get credit, you must show adequate work to justify your answers. If unsure, show the work. No outside materials are permitted on this exam – no notes, papers, books, calculators, phones, smartwatches, or computers – only pens and pencils, and your coursepack. You may use any result in the coursepack (whether boxed or an exercise). However, you must cite it, and you may not use it to prove itself (or a portion of itself). Each problem is out of 10 points, 100 points maximum. You have 75 minutes.

1. Find the set of common divisors of $8, 28$, and use this to find $\gcd(8, 28)$.
2. Find $q, r \in \mathbb{Z}$ so that $(-22, 7) \to DA \to (q, r)$.
3. Let $a, b \in \mathbb{Z}$, not both zero. Set $d = \gcd(a, b)$. Prove that $\gcd(\frac{a}{d}, \frac{b}{d}) = 1$.
4. Use the Euclidean algorithm to find $\gcd(30, 17)$ and also to find $u, v \in \mathbb{Z}$ with $30u + 17v = \gcd(30, 17)$.
5. Let $a, b, c \in \mathbb{Z}$, all nonzero. Suppose that $c|a$ and $c|b$. Prove that $c|\gcd(a, b)$.
6. Let $a, b, c \in \mathbb{Z}$. Suppose that $a|bc$ and that $\gcd(a, b) = 1$. Prove that $a|c$.
7. Let $a, b, c, n \in \mathbb{Z}$ with $n \geq 1$. Suppose that $a \equiv b \pmod{n}$. Prove that $ac \equiv bc \pmod{n}$.
8. Let $a, b, n \in \mathbb{N}$ with $n \geq 1$. Prove that $a \equiv b \pmod{n}$, if and only if $[a] = [b]$.
9. Let $a, n \in \mathbb{Z}$ with $n \geq 2$. Suppose that $[a] = [n - 1]$ modulo n. Prove that $\gcd(a, n) = 1$.
10. Let $a, b \in \mathbb{Z}$ with $\gcd(a, b) = 1$. Prove that, for all positive $n \in \mathbb{Z}$, that $\gcd(a, b^n) = 1$.